**1.** The curve $C$ has equation

$$px^3 + qxy + 3y^2 = 26$$

where $p$ and $q$ are constants.

(a) Show that

$$\frac{dy}{dx} = \frac{apx^2 + bqy}{qx + cy}$$

where $a$, $b$ and $c$ are integers to be found.

**(4)**

Given that

- the point $P(-1, -4)$ lies on $C$
- the normal to $C$ at $P$ has equation $19x + 26y + 123 = 0$

(b) find the value of $p$ and the value of $q$.

**(5)**

a) $px^3 + qxy + 3y^2 = 26$

$\frac{d}{dx}(qxy) = qy + qx\frac{dy}{dx}$ 1

$3px^2 + qy + qx\frac{dy}{dx} + 6y\frac{dy}{dx} = 0$ 1

$3px^2 + qy + \frac{dy}{dx}(qx + 6y) = 0$

$\frac{dy}{dx}(qx + 6y) = -3px^2 - qy$ 1

$\frac{dy}{dx} = \frac{-3px^2 - qy}{qx + 6y}$ 1

b) $P(-1,-4)$ lies on $C$:

$$p(-1)^3 + q(-1)(-4) + 3(-4)^2 = 26 \quad (1)$$

$$-p + 4q + 48 = 26$$

$$-p + 4q = -22 \quad (1)$$

Normal to $C$ at $P$ has equation $19x + 26y + 123 = 0$

$$\Rightarrow y = -\frac{19}{26}x - \frac{123}{26} \quad \therefore m = -\frac{19}{26} \quad (1)$$

$$m_{normal} = \frac{-1}{m_{tangent}}$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=-1 \\ y=-4}} = -\frac{1}{-\frac{19}{26}} = \frac{26}{19}$$

$$\Rightarrow \frac{-3p(-1)^2 - q(-4)}{q(-1) + 6(-4)} = \frac{26}{19} \quad (1)$$

$$\frac{-3p + 4q}{-q - 24} = \frac{26}{19}$$

$$19(-3p + 4q) = 26(-q - 24)$$

$$-57p + 76q = -26q - 624$$

$$624 = 57p - 102q \quad (2)$$

(1)

solve (1) and (2) simultaneously using calculator:

$p = 2 \quad q = -5$

(1)